



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2026

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



- 1 Find the first three terms in the Maclaurin's series for $\sinh^{-1}(x+1)$ in the form $\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 Let $I_n = \int_0^1 (1+t)^n \sin \pi t \, dt$, where n is a non-negative integer.

(a) Show that, for $n \geq 2$, $\pi^2 I_n = \pi(2^n + 1) - n(n-1)I_{n-2}$. [4]

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(b) Find the exact value of I_2 . [3]

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[4]

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(b) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \coth x = \sinh^2 x,$$

given that $y = 0$ when $x = \ln \frac{1}{3}$. Give your answer in the form $y = f(x)$.

[7]





4 The curve C has equation

$$(x-y)^5 + \ln(xy) = 1 + \ln 2.$$

(a) Show that, at the point $(-1, -2)$ on C , $\frac{dy}{dx} = \frac{8}{11}$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.

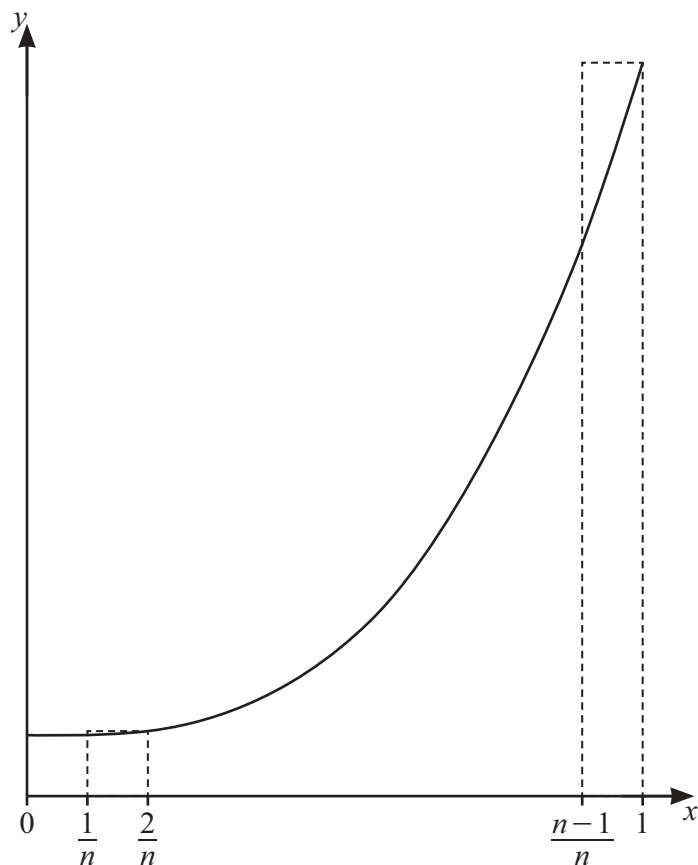


(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$.

[5]



5



The diagram shows the curve with equation $y = \frac{11}{10}x^3 + \frac{1}{10}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10} \right) dx < U_n$, where

$$U_n = \frac{11}{40} \left(1 + \frac{1}{n} \right)^2 + \frac{1}{10}. \quad [4]$$

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- (b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10} \right) dx$. [4]

- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]





6 Find the particular solution of the differential equation

$$5 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + y = x^2 + 7x + 1,$$

given that, when $x = 0, y = 0$ and $\frac{dy}{dx} = 0$. [10]

[illegible]



This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 7 (a) State the sum of the series $z+z^2+z^3+\dots+z^n$, for $z \neq 1$. [1]

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- (b)** Find all roots of the equation $1 + z + z^2 + \dots + z^6 = 0$ in the form $e^{iq\pi}$, where q is rational. [2]

[illegible]

- (c) Given instead that $z = e^{-1+i\theta}$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} e^{-m} \cos m\theta = \frac{e \cos \theta - 1}{e^2 - 2e \cos \theta + 1}. \quad [7]$$

[illegible]

[illegible]

$$by + 2z = 3$$

does not have a unique solution. Find a in terms of b .

[3]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(b) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are integers to be determined.

[4]





(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^7 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.

[7]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

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